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Thermostatistical properties of q-deformed bosons trapped in a D-dimensional power-law potential

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Abstract

The thermostatistical properties of an ideal gas of q-deformed bosons trapped in a D-dimensional power-law potential are studied, based on the q-deformed Bose–Einstein distribution. The effects of q-deformation on the properties of the system are discussed. It is shown that q-deformed bosons ($q \neq 1$) possess many different characteristics from those of ordinary bosons, which include the condition that Bose–Einstein condensation (BEC) occurs, the critical temperature and the continuity of heat capacity.

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1. Introduction

Since the seminal works were introduced and developed by Sklyanin, Kulish, Reshetikhin and Drinfed [1–3], there has been increasing interest in the study of q-deformation and quantum groups [4–20]. For example, a q-deformed boson realization technique was proposed in order to find the representations of quantum groups [4–6]. It has been shown that the entire structure of thermodynamics of q-deformed bosons is preserved, but the ordinary thermodynamics derivative should be replaced by the Jackson derivative (JD) [7–9]. The statistical distribution of q-deformed bosons is found to be different from the standard Bose–Einstein distribution, and thus q-deformed bosons ($q \neq 1$) may possess some different characteristics compared with ordinary bosons.

Several authors have studied the thermostatistical characteristics of an ideal gas of q-deformed bosons in the absence of an external potential [8–12]. However, the properties of trapped q-deformed boson systems, which may be more closely related to the experiments of Bose–Einstein condensation (BEC) in ultracold-trapped Bose gases [21–23], have rarely been investigated. In the present paper, we will study the thermostatistical properties of an ideal gas of q-deformed bosons trapped in a generic power-law potential, based on the q-deformed Bose–Einstein distribution. The general expressions for the critical temperature of BEC, the condensation fraction of particles, the total energy and the heat capacity of the system are derived analytically. The effects of q-deformation on the thermostatistical properties of the system are discussed.

2. The *q*-deformed boson algebra and the distribution of *q*-deformed bosons

The q-deformed boson algebra is realized by defining the q-deformed creation operator $\hat{a}^+$ and annihilation operator $\hat{a}$ which satisfy the commutation relations [4–6]

$$[\hat{N}, \hat{a}^+] = \hat{a}^+ \qquad [\hat{N}, \hat{a}] = -\hat{a} \tag{1}$$

and the nonlinear relations

$$\hat{a}^+\hat{a} = [\hat{N}] \qquad \hat{a}\hat{a}^+ = [\hat{N}+1] \tag{2}$$

where $\hat{N}$ is the number operator, the notation $[x]$ is defined as

$$[x] \equiv \frac{q^x - q^{-x}}{q - q^{-1}} \tag{3}$$

and $q \in \mathbb{R}^+$ is the deformation parameter.

In order to determine the statistical distribution of q-deformed bosons, we choose the Hamiltonian of an ideal q-deformed boson system proposed in [10] as

$$\hat{H}_q = \sum_k (\varepsilon_k - \mu_q)\hat{N}_k \tag{4}$$

where k is the state label, $\hat{N}_k$ and ε_k are, respectively, the number operator and energy of state k and μ_q is the q-deformed chemical potential. The mean value of the q-deformed occupation number $f_{k,q}$ is defined by [10]

$$[f_{k,q}] = \frac{1}{\Xi}\mathrm{tr}\big\{\exp(-\beta\hat{H}_q)\hat{a}_k^+\hat{a}_k\big\} \tag{5}$$

where $\beta = 1/(k_B T)$, k_B is the Boltzmann constant, $\Xi = \mathrm{tr}(\mathrm{e}^{-\beta\hat{H}_q})$ is the partition function. From equations (1), (2), (4) and (5), one can derive

$$\frac{[f_{k,q}]}{[f_{k,q}+1]} = \exp\{-\beta(\varepsilon_k - \mu_q)\}. \tag{6}$$

Using equations (3) and (6), we obtain

$$f_{k,q} = \frac{1}{2\ln q}\ln\frac{z_q^{-1}\exp(\beta\varepsilon_k) - q^{-1}}{z_q^{-1}\exp(\beta\varepsilon_k) - q} \tag{7}$$

where $z_q = \exp(\beta\mu_q)$ is the q-deformed fugacity.

Equation (7) provides the statistical distribution of q-deformed bosons. It may be called the q-deformed Bose–Einstein distribution. It can be seen from equation (7) that $f_{k,q}$ possesses the following properties:

(1) when $q \to 1$, equation (7) becomes

$$f_{k,1} = \frac{1}{z_1^{-1}\exp(\beta\varepsilon_k) - 1} \tag{8}$$

which is just the standard Bose–Einstein distribution. This means that q-deformed bosons will be the same as ordinary bosons when $q \to 1$.

(2) The distribution $f_{k,q}$ should be nonnegative. This gives the following restrictive conditions on the q-deformed fugacity and chemical potential:

$$\begin{cases} z_q \leqslant 1/q & \mu_q \leqslant -k_B T \ln q \qquad (q \geqslant 1) \\ z_q \leqslant q & \mu_q \leqslant k_B T \ln q \qquad (q \leqslant 1). \end{cases} \tag{9}$$

When $q \to 1$, equation (9) is reduced to

$$z_1 \leqslant 1 \qquad \text{or} \qquad \mu_1 \leqslant 0 \tag{10}$$

which are the restriction conditions on the fugacity and chemical potential of an ordinary Bose system.

(3) $f_{k,q}$ possesses the symmetry property [10]

$$f_{k,q} = f_{k,1/q} \tag{11}$$

as long as the condition $z_q = z_{1/q}$ is satisfied (see later).

(4) $f_{k,q}$ can be expanded as

$$f_{k,q} = \frac{1}{2\ln q}\left[\sum_{j=1}^{\infty}(q^j - q^{-j})\frac{z_q^j \exp(-j\beta\varepsilon_k)}{j}\right]. \tag{12}$$

It should be pointed out that the definitions of the notation $[x]$, Hamiltonian $\hat{H}_q$ and mean value of the occupation number $f_{k,q}$ in literature are not unique. For example, the notation $[x]$ in [13, 14] is given by a non-symmetric form such as $[x] \equiv (q^x - 1)/(q - 1)$; the authors in [17, 24, 25] introduced different Hamiltonians which involve $[\hat{N}_k] = \hat{a}_k^+ \hat{a}_k$ [17] or $\left(\hat{a}_k^+\hat{a}_k + \hat{a}_k\hat{a}_k^+\right)/2$ [24, 25] instead of $\hat{N}_k$; the mean value of the occupation number in [25, 26] is defined by replacing $[f_{k,q}]$ in equation (5) with $f_{k,q}$. It is shown that different choices of $[x]$, $\hat{H}_q$ and $f_{k,q}$ may result in different distribution functions and, consequently, different dependences of the properties of q-deformed systems on the parameter q [8–12, 24–26]. All the results derived in this paper are based on equations (3)–(5), which give the definitions of $[x]$, $\hat{H}_q$ and $f_{k,q}$, respectively.

3. Thermostatistical properties of trapped q-deformed bosons

Now, let us consider an ideal gas of q-deformed bosons trapped in a D-dimensional power-law potential. The energy spectrum of a single particle is

$$\varepsilon = ap^s + \sum_{i=1}^{D} U_i \left|\frac{x_i}{L_i}\right|^{t_i} \tag{13}$$

where a, s, L_i, U_i and t_i are all positive constants, p and x_i are, respectively, the momentum and the ith component of coordinate of a particle. It is seen that the potential is very general, which includes a variety of particular potentials corresponding to the different values of L_i, U_i and t_i.

The total number of particles and total energy of the system are, respectively, given by

$$N = \sum_k f_{k,q} \tag{14}$$

$$E = \sum_k \varepsilon_k f_{k,q}. \tag{15}$$

For the system of a large number of particles, the sum over state k may be replaced by the integral over the phase space. Thus, according to equations (12) and (13), equations (14) and (15) can be, respectively, expressed as

$$N = N_0 + \frac{g}{2h^D \ln q} \sum_{j=1}^{\infty} (q^j - q^{-j}) \frac{z_q^j}{j} \int \exp\left[-j\beta\left(ap^s + \sum_{i=1}^{D} U_i \left|\frac{x_i}{L_i}\right|^{t_i}\right)\right] \prod_{i=1}^{D} \mathrm{d}p_i\, \mathrm{d}x_i$$

$$= N_0 + \frac{gC_D \Gamma(D/s+1) g_{\eta+1,q}(z_q)(k_\mathrm{B}T)^{\eta} \prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}{h^D a^{D/s} \prod_{i=1}^{D} U_i^{1/t_i}} \tag{16}$$

$$E = \frac{g}{2h^D \ln q} \sum_{j=1}^{\infty} (q^j - q^{-j}) \frac{z_q^j}{j} \int \left(ap^s + \sum_{i=1}^{D} U_i \left|\frac{x_i}{L_i}\right|^{t_i}\right)$$

$$\times \exp\left[-j\beta\left(ap^s + \sum_{i=1}^{D} U_i \left|\frac{x_i}{L_i}\right|^{t_i}\right)\right] \prod_{i=1}^{D} \mathrm{d}p_i\, \mathrm{d}x_i$$

$$= \frac{gC_D \Gamma(D/s+1)\eta g_{\eta+2,q}(z_q)(k_\mathrm{B}T)^{\eta+1} \prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}{h^D a^{D/s} \prod_{i=1}^{D} U_i^{1/t_i}} \tag{17}$$

where

$$N_0 = \frac{1}{2\ln q} \ln \frac{z_q^{-1} - q^{-1}}{z_q^{-1} - q} \tag{18}$$

is the number of particles in the ground state,

$$g_{l,q}(z_q) = \frac{1}{2\ln q} \sum_{j=1}^{\infty} (q^j - q^{-j}) \frac{z_q^j}{j^l} \tag{19}$$

may be called the q-deformed Bose integral, $C_D = \pi^{D/2}/\Gamma(D/2+1)$ is the volume of a D-dimensional sphere with unit radius, g is the spin degenerate factor, h is the Planck constant, $\Gamma(l) = \int_0^\infty x^{l-1} \exp(-x)\,\mathrm{d}x$ is the gamma function and $\eta \equiv D/s + \sum_{i=1}^{D} 1/t_i$. It can be seen from equation (19) that there exists the following relation between $g_{l,q}(z_q)$ and the ordinary Bose integral $g_l(z_1)$:

$$\lim_{q\to 1} g_{l,q}(z_q) = g_{l-1}(z_1). \tag{20}$$

From equations (16), (18) and (19) we see that from the form of N and T, it is a natural requirement that $z_{1/q} = z_q$, when q is replaced by $1/q$. It is clear that equation (11) is true. This implies that the thermostatistical properties of q-deformed bosons will remain unchanged under the transformation of $q \leftrightarrow 1/q$. Thus, we shall restrict the following discussions to $q \geqslant 1$.

It can be found from equation (16) that when $z = 1/q$ and the particle number in the ground state is still macroscopically negligible, i.e. $N_0 = 0$,

$$T = \frac{1}{k_\mathrm{B}} \left[\frac{Nh^D a^{D/s} \prod_{i=1}^{D} U_i^{1/t_i}}{gC_D \Gamma(D/s+1)\zeta_q(\eta+1) \prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}\right]^{1/\eta} = T_0 \left[\frac{1}{\zeta_q(\eta+1)}\right]^{1/\eta} \equiv T_{c,q} \tag{21}$$

is referred to as the critical temperature of BEC of q-deformed bosons, where

$$T_0 = \frac{1}{k_\mathrm{B}} \left[\frac{Nh^D a^{D/s} \prod_{i=1}^{D} U_i^{1/t_i}}{gC_D \Gamma(D/s+1) \prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}\right]^{1/\eta} \tag{22}$$

and

$$\zeta_q(l) \equiv g_{l,q}(1/q) \tag{23}$$

may be called the q-deformed Riemann zeta function. It is easily proved that the relation between $\zeta_q(l)$ and the Riemann zeta function $\zeta(l)$ is as follows:

$$\lim_{q\to 1} \zeta_q(l) = \zeta(l-1). \tag{24}$$

From equations (16) and (21), one can find that when $T \leqslant T_{c,q}$, the fraction of particles in the ground state is given by

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_{c,q}}\right)^{\eta}. \tag{25}$$

The heat capacity with a given external potential and particle number is $C = \partial E/\partial T$. When $T > T_{c,q}$, we can derive

$$C_{T>T_{c,q}} = Nk_{\mathrm{B}}\left[\eta(\eta+1)\frac{g_{\eta+2,q}(z_q)}{g_{\eta+1,q}(z_q)} - \eta^2\frac{g_{\eta+1,q}(z_q)}{g_{\eta,q}(z_q)}\right] \tag{26}$$

from equations (16) and (17). When $T < T_{c,q}$, $z_q = 1/q$ and the total energy may be expressed as

$$E_{T<T_{c,q}} = \frac{gC_D\Gamma(D/s+1)\eta\zeta_q(\eta+2)(k_{\mathrm{B}}T)^{\eta+1}\prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}{h^D a^{D/s}\prod_{i=1}^{D}U_i^{1/t_i}}. \tag{27}$$

Using equations (21) and (27), we get

$$C_{T<T_{c,q}} = Nk_{\mathrm{B}}\eta(\eta+1)\frac{\zeta_q(\eta+2)}{\zeta_q(\eta+1)}\left(\frac{T}{T_{c,q}}\right)^{\eta}. \tag{28}$$

From equations (26) and (28), it can be found that the jump of the heat capacity between $T \to T_{c,q}^-$ and $T \to T_{c,q}^+$ is

$$\Delta C = C_{T\to T_{c,q}^-} - C_{T\to T_{c,q}^+} = Nk_{\mathrm{B}}\eta^2\frac{\zeta_q(\eta+1)}{\zeta_q(\eta)}. \tag{29}$$

4. Discussion

Equations (16), (21), (25), (26), (28) and (29) are the main results in the present paper. Starting from these equations, we will further deduce some new thermostatistical characteristics of the system.

(1) Equation (16) can be expressed as

$$N = N_0 + \frac{gV_D^*}{\lambda^D} g_{\eta+1,q}(z_q) \tag{30}$$

where

$$V_D^* = \prod_{i=1}^{D}(2L_i)\prod_{i=1}^{D}\Gamma\left(\frac{1}{t_i}+1\right)\left(\frac{k_{\mathrm{B}}T}{U_i}\right)^{1/t_i} \tag{31}$$

and

$$\lambda = \frac{ha^{1/s}}{\pi^{1/2}(k_{\mathrm{B}}T)^{1/s}}\left[\frac{\Gamma(D/2+1)}{\Gamma(D/s+1)}\right]^{1/D} \tag{32}$$

are, respectively, the effective volume and thermal wavelength of the system [27]. Equation (30) shows that the value of $g_{\eta+1,q}(z_q)$ is independent of q for a given value of

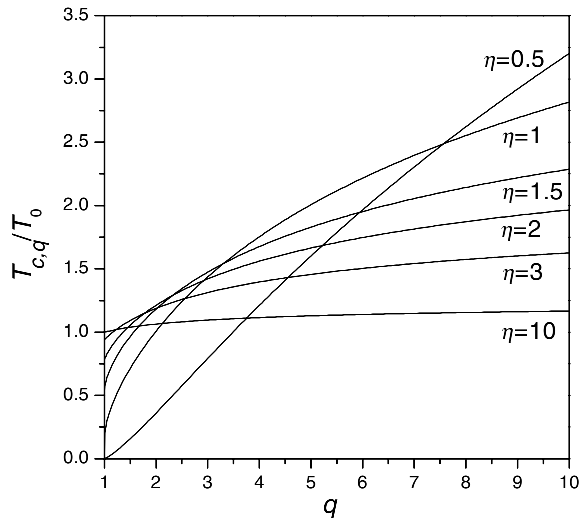

Figure 1. The critical temperature of q-deformed bosons as a function of q for different η.

the temperature $T > T_{c,q}$, since V_D^* and λ are both independent of q. This indicates that when $T > T_{c,q}$, z_q should be such a function of q that $g_{\eta+1,q}(z_q)$ is the same for various values of q.

(2) When $q \neq 1$, $\zeta_q(\eta + 1)$ is convergent for any value of $\eta > 0$. Thus, according to equation (21), BEC may occur for q-deformed bosons ($q \neq 1$) trapped in an arbitrary dimensional space and in any external potentials of power-law form. The result is different from that for ordinary bosons ($q = 1$), which have been proved to be impossible to condense when $\eta \leqslant 1$ [28].

Figure 1 shows the $T_{c,q} \sim q$ curves for some different η. It can be seen from figure 1 that $T_{c,q}$ always increases with q. The increasing rates, however, are different for different η. The smaller η is, the larger the influence of q on the critical temperature may be. When η is small and q is far away from 1, $T_{c,q}$ may be much higher than $T_{c,1}$. For the q-deformed boson systems with a large η, however, the increase of $T_{c,q}$ with q is very slow, so that $T_{c,q}$ may be very close to $T_{c,1}$. This shows clearly that the influence of q-deformation on the thermostatistical properties of the system is small when η is large.

(3) From equation (25), one can find that when $T \leqslant T_{c,q}$,

$$\left|\frac{\mathrm{d}N_0}{\mathrm{d}T}\right| = \frac{\eta N}{T_{c,q}}\left(\frac{T}{T_{c,q}}\right)^{\eta-1}. \tag{33}$$

Equation (33) shows that when $\eta > 1$, the condensing rate of particles in the ground state $|\mathrm{d}N_0/\mathrm{d}T|$ decreases from $N\eta/T_{c,q}$ to zero when the temperature is decreased from $T_{c,q}$ to zero and the $N_0/N \sim T/T_{c,q}$ curve is concave. However, when $\eta < 1$, $|\mathrm{d}N_0/\mathrm{d}T|$ increases from $N\eta/T_{c,q}$ to infinite when the temperature is decreased from $T_{c,q}$ to zero and the $N_0/N \sim T/T_{c,q}$ curve is convex. When $\eta = 1$, $|\mathrm{d}N_0/\mathrm{d}T|$ remains constant for all $T \leqslant T_{c,q}$ and the $N_0/N \sim T/T_{c,q}$ curve is a straight line. Figure 2 shows the three curves mentioned above. It should be pointed out that the result obtained here is quite different from that of ordinary bosons whose $N_0/N \sim T/T_{c,1}$ curves can only be concave [28].

(4) Equation (29) shows that when $\eta \leqslant 1$, $\Delta C = 0$ because $\zeta_q(\eta)$ is divergent. This implies the fact that the heat capacity is continuous at $T_{c,q}$. When $\eta > 1$ and $q \neq 1$, $\Delta C \neq 0$ because $\zeta_q(\eta)$ is convergent. This implies the other fact that the heat capacity is discontinuous

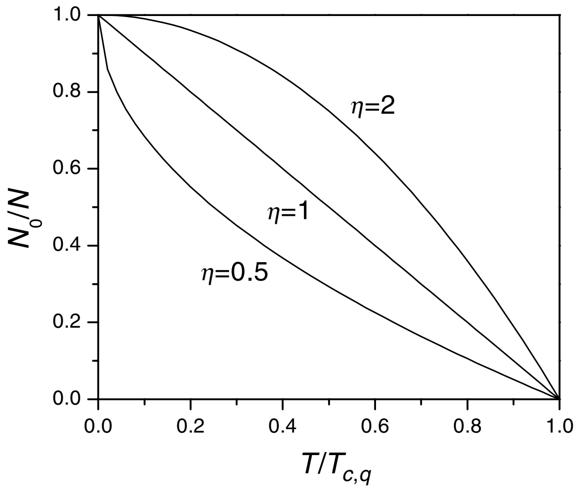

Figure 2. The ground state fraction of particles as a function of temperature for different η.

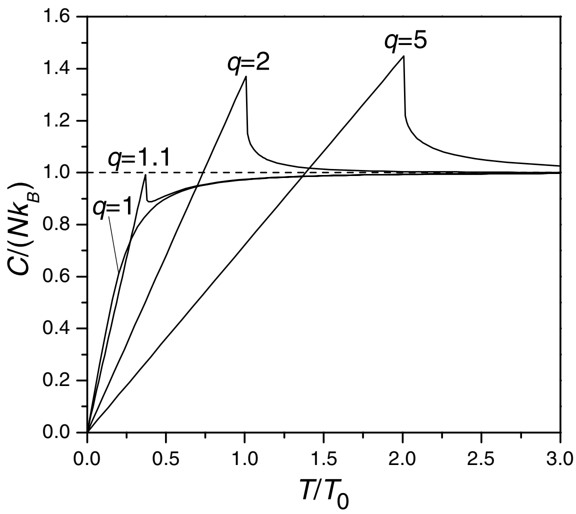

Figure 3. The heat capacity as a function of temperature for a q-deformed boson system of $\eta = 1$.

at $T_{c,q}$. The result gives rise to the second obvious difference between q-deformed bosons ($q \neq 1$) and ordinary bosons, because the heat capacity of the latter is continuous at $T_{c,1}$ when $\eta \leqslant 2$ and discontinuous at $T_{c,1}$ when $\eta > 2$ [28].

Using equations (16), (26) and (28) and the numerical calculation, one can expound on the dependence of the heat capacity on the temperature. The curves of the heat capacity varying with dimensionless temperature T/T_0 for the systems of $\eta = 1$ and $\eta = 1.5$ are shown in figures 3 and 4, respectively. It is seen from figure 3 that, for the system of $\eta = 1$, the heat capacity is continuous but a turning point exists at $T_{c,q}$. The value of C at $T_{c,q}$ decreases with decreasing q. When $q \to 1$, $C_{T=T_{c,q}} \to 0$ and the heat capacity increases smoothly with increasing temperature. The result is quite natural, because $T_{c,q} \to 0$ when $q \to 1$. In this case, no BEC occurs. From figure 4 one can find that, for the system of $\eta = 1.5$, there exists

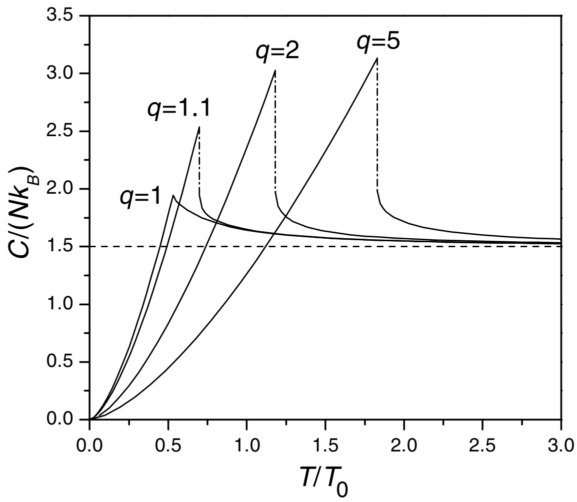

Figure 4. The heat capacity as a function of temperature for a q-deformed boson system of $\eta = 1.5$.

a jump of the heat capacity ΔC between $T \rightarrow T_{c,q}^{-}$ and $T \rightarrow T_{c,q}^{+}$, which is determined by equation (29). It is found that ΔC becomes small when q comes close to 1, and $\Delta C = 0$ when $q \rightarrow 1$. The result is in accordance with the fact that the heat capacity is continuous for the ordinary boson system of $\eta = 1.5$, which corresponds to the case of a boson system with $D = 3$, $s = 2$ and $t_i \rightarrow \infty$ $(i = 1, 2, 3)$.

Another characteristic appearing in figures 3 and 4 is that for different q, the heat capacities at high temperatures converge towards the same values as those for ordinary bosons, which are Nk_{B} and 1.5 Nk_{B} for $\eta = 1$ and $\eta = 1.5$, respectively. In fact, at high temperatures, not only the heat capacity but also other properties of q-deformed bosons will become the same as those of ordinary bosons. One can reach the conclusion easily by analysing the behaviour of the distribution function at high temperatures. When $T \gg T_{c,q}$, $z_q \ll 1/q$ and then equation (12) may be written as

$$f_{k,q} \approx \frac{q - q^{-1}}{2\ln q} z_q \exp(-\beta\varepsilon_k) = \frac{N}{Z} \exp(-\beta\varepsilon_k) \tag{34}$$

where $N = \sum_k f_{k,q}$ is the total number of particles and $Z = \sum_k \exp(-\beta\varepsilon_k)$ is the partition function of a single particle. Equation (34) is just the result of the Boltzmann distribution, which is independent of q.

(5) The results of the present paper are very general and can be used to derive the thermostatistical properties of some particular q-deformed boson and ordinary boson systems. For example, when $s = 2$, $a = 1/(2m)$ and $t_i \rightarrow \infty$ $(i = 1, 2, \ldots, D)$, equations (21), (26) and (28) are reduced to

$$T_{c,q} = \frac{h^2}{2\pi m k_{\mathrm{B}}} \left[\frac{N}{g V_D \zeta_q(D/2+1)}\right]^{2/D} \tag{35}$$

$$C_{T>T_{c,q}} = Nk_{\mathrm{B}} \left[\frac{D}{2}\left(\frac{D}{2}+1\right)\frac{g_{D/2+2,q}(z_q)}{g_{D/2+1,q}(z_q)} - \frac{D^2}{4}\frac{g_{D/2+1,q}(z_q)}{g_{D/2,q}(z_q)}\right] \tag{36}$$

$$C_{T<T_{c,q}} = Nk_{\mathrm{B}} \frac{D}{2}\left(\frac{D}{2}+1\right)\frac{\zeta_q(D/2+2)}{\zeta_q(D/2+1)}\left(\frac{T}{T_{c,q}}\right)^{D/2} \tag{37}$$

respectively. They are just the critical temperature and heat capacity of nonrelativistic q-deformed bosons confined in a D-dimensional box of volume $V_D = \prod_{i=1}^{D}(2L_i)$, which has been discussed in [12]. When $s = 2$, $a = 1/(2m)$ and $t_i = 2$ and $U_i\big/L_i^2 = m\omega^2/2\,(i = 1, 2, \ldots, D)$, equations (21), (26) and (28) will give the expressions for the critical temperature and heat capacity of nonrelativistic q-deformed bosons trapped in an isotropic harmonic potential with frequency ω. When $q \to 1$, equations (21), (26) and (28) can be, respectively, expressed as

$$T_{c,1} = \frac{1}{k_{\mathrm{B}}}\left[\frac{Nh^D a^{D/s}\prod_{i=1}^{D}U_i^{1/t_i}}{gC_D\Gamma(D/s+1)\zeta(\eta)\prod_{i=1}^{D}(2L_i)\Gamma(1/t_i+1)}\right]^{1/\eta} \tag{38}$$

$$C_{T>T_{c,1}} = Nk_{\mathrm{B}}\left[\eta(\eta+1)\frac{g_{\eta+1}(z_1)}{g_\eta(z_1)} - \eta^2\frac{g_\eta(z_1)}{g_{\eta-1}(z_1)}\right] \tag{39}$$

$$C_{T<T_{c,1}} = Nk_{\mathrm{B}}\eta(\eta+1)\frac{\zeta(\eta+1)}{\zeta(\eta)}\left(\frac{T}{T_{c,1}}\right)^{\eta}. \tag{40}$$

They are, respectively, the expressions for the critical temperature and heat capacity of an ordinary boson system trapped in a D-dimensional power-law potential [28, 29]. If it is further assumed that $t_i \to \infty\,(i = 1, 2, \ldots, D)$, $D = 3$, $s = 2$ or 1 and $a = 1/2m$ or c, the properties of a nonrelativistic or ultrarelativistic ordinary boson system confined in a three-dimensional box, which have been discussed in many textbooks [30, 31], can be directly derived from the above results.

It is of significance to note that the idea of q-deformation involves more than just modifying the already available physical models mathematically [32]. It not only gives a better chance of understanding the peculiarities of undeformed theory, but also reveals the characteristics of some ordinary quantum systems which may contain the q-deformed structures inherently in themselves [33–35]. Recent investigations have revealed that the theory of q-deformation may provide possible applications in a variety of areas, such as anyon physics [36, 37], vertex models [38] and quantum mechanics in discontinuous spacetime [39], etc. One of the main aims of the present paper is to introduce the quantum group symmetries in the thermodynamic system and focus on the effects of q-deformation on the thermostatistical properties of a trapped Bose gas. The results obtained here may be helpful to us for understanding further the properties of a trapped Bose gas and for investigating the connection between the parameter q and other parameters of the system.

5. Conclusions

We have studied the thermostatistical properties of a q-deformed boson system trapped in a D-dimensional power-law potential, based on the q-deformed Bose–Einstein distribution. Several important physical quantities of the system are derived analytically. It is shown that many properties of q-deformed bosons ($q \neq 1$), such as the condition that BEC occurs, the critical temperature, the $N_0/N \sim T/T_{c,q}$ curves and the continuity of the heat capacity at the critical temperature, are different from those of ordinary bosons. However, these differences will vanish when $q \to 1$.

Although only one q-deformed boson system trapped in an external potential is studied, the general expressions derived in the present paper can be used to explore the thermostatistical properties of a variety of q-deformed boson and ordinary boson systems, such as q-deformed boson systems confined in a box, q-deformed boson systems trapped in a harmonic potential and the systems of nonrelativistic or ultrarelativistic ordinary bosons and so on. Therefore, the results obtained here are general and useful.

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References

[1] Sklyanin E K 1982 *Funct. Anal. Appl.* **16** 262
[2] Kulish P P and Reshetikhin N Y 1983 *J. Sov. Math.* **23** 2435
[3] Drinfeld V G 1988 *Sov. Math. Dokl.* **36** 212
[4] Macfarlane A J 1989 *J. Phys. A: Math. Gen.* **22** 4581
[5] Biedenharn L C 1989 *J. Phys. A: Math. Gen.* **22** L873
[6] Sun C P and Fu H C 1989 *J. Phys. A: Math. Gen.* **22** L983
[7] Jackson F 1909 *Mess. Math.* **38** 57
[8] Lavagno A and Swamy P N 2000 *Phys. Rev.* E **61** 1218
[9] Lavagno A and Swamy P N 2002 *Chaos Solitons Fractals* **13** 437
[10] Tuszynski J A, Rubin J L, Meyer J and Kibler M 1993 *Phys. Lett.* A **175** 173
[11] Ubriaco M R 1998 *Phys. Rev.* E **57** 179
[12] Shu Y, Chen J and Chen L 2002 *Phys. Lett.* A **292** 309
[13] Arik M and Coon D D 1976 *J. Math. Phys.* **17** 524
[14] Arik M, Kornfilt J and Yildiz A 1997 *Phys. Lett.* A **235** 318
[15] Erzan A and Eckmann J P 1997 *Phys. Rev. Lett.* **78** 3245
[16] Pan H and Zhao Z S 2001 *Phys. Lett.* A **282** 251
[17] Gruver J L 1999 *Phys. Lett.* A **254** 1
[18] Zhang J 2000 *Phys. Lett.* B **477** 361
[19] Sogami I S and Koizumi K 2002 *Prog. Theor. Phys.* **107** 1
[20] Lavagno A and Swamy P N 2002 *Phys. Rev.* E **65** 036101
[21] Anderson M H, Ensher J R, Matthews M R, Wieman C E and Cornell E A 1995 *Science* **269** 198
[22] Bradley C C, Sackett C A, Tollett J J and Hulet R G 1995 *Phys. Rev. Lett.* **75** 1687
[23] Davis K B, Mewes M O, Andrew M R, van Druten N J, Durfee D S, Kurn D M and Ketterle W 1995 *Phys. Rev. Lett.* **75** 3969
[24] Su G and Ge M L 1993 *Phys. Lett.* A **173** 17
[25] Ge M L and Su G 1991 *J. Phys. A: Math. Gen.* **24** L721
[26] Gong R S 1995 *Phys. Lett.* A **199** 81
[27] Yan Z 2000 *Phys. Rev.* A **61** 063607
[28] Yan Z 1999 *Phys. Rev.* A **59** 4657
[29] Yan Z, Li M, Chen L, Chen C and Chen J 1999 *J. Phys. A: Math. Gen.* **32** 4069
[30] Huang K 1987 *Statistical Mechanics* 2nd edn (New York: Wiley)
[31] Pathria R K 1977 *Statistical Mechanics* (Oxford: Pergamon)
[32] Abe S 1998 *Phys. Lett.* A **244** 229
[33] Abe S 1996 *Phys. Rev.* A **54** 93
[34] Celeghini E, Martino S D, Siena S D, Rasetti M and Vitiello G 1995 *Ann. Phys., NY* **241** 50
[35] Chen G H, Kuang L M and Ge M L 1996 *Phys. Lett.* A **213** 231
[36] Caracciolo R and Monteiro M R 1993 *Phys. Lett.* B **308** 58
[37] Lerda A and Sciuto S 1993 *Nucl. Phys.* B **401** 613
[38] Witten E 1990 *Nucl. Phys.* B **330** 285
[39] Dimakis A and Müller-Hoissen F 1992 *Phys. Lett.* B **295** 242